EXISTENCE AND CONSTRUCTION OF OPTIMAL COMBINATORIAL t- DESIGNS

A synopsis for the proposed Ph.D. Research work in Mathematics

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1. Title: Existence and Construction of Optimal Combinatorial t-Designs.

2. State of the Art:

2.1 Origin of Design:

Design theory has been in practice since around 1600 B.C. The Study of Design theory and their strength are recent developments. This study is called Block Design in general. In 1850, T. Kirkman solved the following problem, originally proposed by W. S. B. Woolhouse in 1844, A school mistress is in the habit of taking 15 girls of her school for a daily morning walk in 5 batches of 3 girls each, so that each girl has 2 companions. Is it possible to find an arrangement so that for 7 consecutive days, no girl walks with any of her companions in any batch more than once?

At this point of time design theory has a complex structure. To understand it needs some basic terms defined, which are mentioned below.

2.2 Some terms used in Block Design:

Design: A design is a pair (V, B), where V is a finite set of point(Varieties) and B is a collection of subsets of V, called **blocks** of the design.

t-Design: A t-($v,\ k$, λ) design is a design on v points with blocks of cardinality $k,$ such that every t-subset of V appears in exactly λ blocks.

BIBD: A 'BIBD' or 2-(v, k, λ) design is an arrangement of v treatments in 'b' blocks, each size $k(< v)$ such that every treatment occurs in exactly r blocks and any two distinct treatments occur together in λ blocks. The integers b, v, r, k, λ are called parameters of the BIBD.

Resolvable Design: A block design of b blocks in which each of v treatments is replicated 'r' times is said to be resolvable if 'b' blocks can be divided into 'r' sets of $\frac{b}{r}$.

Near resolvable Design: A block design D (b, v, r, k) is said to be near resolvable if its blocks can be partitioned into v classes such that for each treatment θ of D there is precisely one class which does not contain θ in any of its blocks and each class $v-1$ distinct treatments.

Group Divisible Design: Let $v = mn$ treatments be arranged in an $m \times n$ array. A group divisible design is an arrangement of the $v = mn$ treatments in b blocks each of size k such that:

1. Every treatment occurs at most once in a block;
2. Every treatment occurs in r blocks;
3. Every pair of treatments, which are in the same row of the $m \times n$ array, occurs together in λ_1 blocks; while every other pair of treatment occurs together in λ_2 blocks.

2.3 Recent Advances in t- Design:

In the year 2021, R. J. R. Abel et al. [7] proposed existence of 4-GDDs with at most 50 points and 4-GDDs of types 6^s3^t and 9^s3^t**.** In this paper, they give two new refinements to the known necessary conditions for the existence of a 4-GDD and list all the feasible group types for 4-GDDs where the number of points v satisfies $31 \leq v \leq 50$. For v in this range, they obtain solutions for all feasible group types when $\boldsymbol{v \equiv 0}$ (mod3), and solutions for all feasible types of the form $\boldsymbol{4^s1^tw^1}$ except $\boldsymbol{4^81^110^1}$. They also obtain various other 4-GDDs. These include solutions for 4-GDDs of types 6^s3^t and 9^s3^t for all but a finite number of feasible values of s and t.

In the year 2022, S. Saurabh [8] constructed difference matrices and generalized weighing matrices from certain BIBDs. In this scheme difference matrices and generalized weighing matrices are obtained from affine resolvable and near-resolvable designs. These matrices are used in the construction of group divisible designs and Latin-square type designs which are important families of two associate classes of partially balanced incomplete block designs.

In the year 2022, T. V. Trung [9] proposed a method of constructing 2-resolvable t-designs for $\mathrm{t} = \boldsymbol{3, 4}$. This construction is based on the assumption that there exists a partition of a t-design into Steiner 2-designs. When t = 4, partitioning a 4-design into projective planes. If the order of projective planes is odd, it also enables to construct 3-resolvable 5-designs with a largest possible block size. In general, the method appears to be very effective, as it helps produce infinitely many 2-resolvable 3-designs with a large variety of block sizes.

In the year 2023, Saurabh and Sinha [10] proposed a Regular group divisible design and Generalised Bhaskar Rao designs using a Symmetric group. In this article, a regular group divisible design with certain parameters is obtained using a balanced generalized Weighing matrix over a dihedral group of order 6.

In the year 2023, R. J. R. Abel et al. [11] proposed a group divisible design with block size where the group sizes are congruent to 1(mod3). In this paper, they consider the existence of 4-GDDs on v points with $v \leq 50$. We consider the case $v \equiv 1$ (mod3) and construct designs for all feasible group types here. We also enumerate all 4-GDDs with type 7^4 and compare the actions of their automorphism groups. When $v \equiv 0$ (mod3) and $v \leq 50$, there are no feasible group types for which existence has not been determined, but for $v \equiv 2$ (mod3) and $v \leq 50$, we obtain 4-GDDs for two more feasible group types.

In spite of the diligent efforts of many researchers 2-(51, 6, 1), 2-(61, 6, 1), 2-(40, 10, 3), 2-(46, 10, 3), 2-(39, 13, 6), 2-(100, 12, 2), 3-(16, 7, 5), 3-(17, 7, 7) , 3-(19, 9, 140), 3-(19, 9, 664), 3-(19, 9, 980), 3-(19, 9, 1484) and many other are still not resolved.

3. Problem to be Investigated:

In this proposed research, we aim

a) To construct new resolvable/ near resolvable t-designs and group divisible t-designs.

b) To analyze of properties of recently discovered t-designs and to discover any possible application of t-designs.

4. The Objective of the Research:

In this era, t-design can reduce the cost and time of conducting experiments, as well as improve the efficiency and precision of the analysis. The objective of this research work is to contribute to this quest.

5. Tools to be used:

This research work will depend on the recent developments in t- Design. For this, relevant books and journals are needed. To develop and test the t-design, mathematical software like MATLAB, Python or others will be used.

6. Contribution to the existing knowledge:

Various mathematical objects have been used to develop error-correcting code but, we aim to employ some combinatorial matrices, which are in the recent trend. Therefore this research work will surely contribute to the existing knowledge.

7. Need of the society:

The society needs help to reduce the cost and time of conducting experiments, as well as improve the efficiency and precision of the analysis. Proposed research work aims to help the society in this area.

8. Conclusion:

This proposed research work will go through extend of recent developments as well as try to explore new ideas that can be contribute to the betterment of society.

References :

[1] R. Mathon and A. Rosa, Tables of Parameters of BIBDs including Existence, Enumeration, and Resolvability Results, *Annals of Discrete Mathematics*, 26, 1985, 275-308.

[2] W. H. Clatworthy, PBIBDs with two associate classes and two treatments per block, *Journal of Research of the National Bureau of Standards*, 54 (4), 1995, 177-190.

[3] R. Mathon and A. Rosa, *2-(v, k, λ) Designs of small Order,* Handbook of Combinatorial Design, 2nd Edition, Chapman & Hall/CRC, 2007.

[4] G. B. Khosrovshahi and R. Laue, *t-Designs with* $t \geq 3$, Handbook of Combinatorial Design, 2nd Edition, Chapman & Hall/CRC, 2007.

[5] R. J. R. Abel, G. Ge and J. Yin, *Resolvable and Near-Resolvable Designs*, Handbook of Combinatorial Design, 2nd Edition, Chapman & Hall/CRC, 2007.

[6] A. Dey, *Incomplete Block Designs*, World Scientific Publishing Co. Pte. Ltd. Singapore, 2010.

[7] R. J. R. Abel, Y. A. Bunjamin and D. Combe, Existence of 4-GDDs with at most 50 points and 4-GDDs of types 6^s3^t and 9^s3^t, *Discrete Mathematics*, 334 (9), 2021, 1-26.

[8] S. Saurabh, Combinatorial matrices from certain BIBDs, *Journal of Statistics and Computer Science*, 1(2), 2022, 159-164.

[9] T.V. Trung, Construction of 2- resolvable t-designs for $t = 3,4$, *Designs, Codes and Cryptography*, 90, 2022, 1567-1583.

[10] S. Saurabh and K. Sinha, Regular Group Divisible Designs Using Symmetric Groups, *Statistics and Application*, 21(1), 2023, 23-26.

[11] R. J. R. Abel, T. Britz, Y. A. Bunjamin, D. Combe, T. Feng, Group divisible design with block size where the group sizes are congruent to 1(mod3), *Discrete Mathematics*, 346(4), 2023, 113277.

Research Guide

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